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Structural FECM: Cointegration in large-scale structural FAVAR models*

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Abstract

Starting from the dynamic factor model for non-stationary data we derive the factor-augmented error correction model (FECM) and its moving-average representation. The latter is used for the identification of structural shocks and their propagation mechanisms. We show how to implement classical identification schemes based on long-run restrictions in the case of large panels. The importance of the error-correction mechanism for impulse response analysis is analysed by means of both empirical examples and simulation experiments. Our results show that the bias in estimated impulse responses in a FAVAR model is positively related to the strength of the error-correction mechanism and the cross-section dimension of the panel. We observe empirically in a large panel of US data that these features have a substantial effect on the responses of several variables to the identified permanent real (productivity) and monetary policy shocks.

Keywords: Dynamic Factor Models, Cointegration, Structural Analysis, Factor-augmented Error Correction Models, FAVAR

JEL-Codes: C32, E17

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1 Introduction

Large dimensional factor models have received considerable attention in the recent econometric literature, starting with the seminal papers by Forni, Hallin, Lippi and Reichlin (2000) and Stock and Watson (2002a, 2002b). While the early applications were mostly reduced form analyses, following the publication of Bernanke, Boivin and Eliasz (2005) more and more attention has been devoted to structural analyses based on Factor Augmented VARs (FAVARs) - see also Stock and Watson (2005).

With few notable exceptions, such as Bai (2004), Bai and Ng (2004) and Barigozzi, Lippi and Luciani (2016a, 2016b), this entire literature does not take into account the possibility of cointegration among the variables under study. Banerjee and Marcellino (2009) suggested including factors extracted from large non-stationary panels in small scale error correction models (ECMs) to proxy for the missing cointegration relations. They labelled the resulting model as the Factor Augmented ECM (FECM). Banerjee, Marcellino and Masten (2014a) showed that FECMs often outperform both FAVARs and standard small scale ECMs in terms of forecasting macroeconomic variables, given the property that FECMs nest both FAVARs and ECMs.

In this paper we focus on the use of FECMs for structural analysis. We start from a dynamic factor model for nonstationary data as in Bai (2004) and Bai and Ng (2004), and show it can be reparameterized to yield a FECM. We then extend the Johansen version of the Granger representation theorem (see, e.g., Johansen, 1995, Theorem 4.2) to derive the moving-average representation of the FECM. The latter can be used to identify structural shocks and their propagation mechanism, using similar techniques as those adopted in the structural VAR literature. In particular, our paper provides the first analysis of the long-run scheme for identification of structural shocks in nonstationary panels.[1]

When assessing the properties of the FECM with respect to the FAVAR, we focus on the effects that including the error-correction terms have on the impulse response functions. Using simulation experiments with a design similar to the estimated model in the empirical applications, we consider which features increase the bias in the impulse responses of the FAVAR with respect to those from the FECM. Not surprisingly, the strength of the error-correction mechanism matters. Moreover, as we show in the paper, since the FECM can be approximated to some extent by the FAVAR with a large lag order, over-parameterization and the associated estimation uncertainty also play a role.

Finally, we develop two empirical applications. The first one uses our proposed long-run restrictions to identify structural stochastic trends and the effects of their associated shocks on a large set of US economic variables. The FECM impulse responses are in

[1]Forni, Giannone, Lippi and Reichlin (2009) provide an empirical illustration of the stochastic trends analysis of King, Plosser, Stock and Watson (1991) in the context of large stationary panels. Eickmeier (2009) works with a nonstationary panel and identification of structural shocks with sign restrictions. Forni, Gambetti and Sala (2014) also consider a factor model for nonstationary data. The FECM model is also related to the framework used recently to formulate testing for cointegration in panels (see for example Bai, Kao and Ng (2009) and Gengenbach, Urbain and Westerlund (2015).

line with economic theory and comparable to the responses to permanent productivity shocks obtained from an estimated DSGE model (Adolfson, Laseen, Linde and Villani, 2007), which validates our identification scheme. The second application is to the analysis of monetary policy shocks, similar to Bernanke et al. (2005). In both cases our results indicate important effects of omitting the error-correction terms in the FAVAR. According to the Olivei and Tenreyro (2010) test, about 30% of the FECM impulse responses result to be statistically significantly different from FAVAR responses in case of a permanent productivity shock, while the corresponding share for the monetary policy shock is above 40%.

The rest of the paper is structured as follows. In Section 2 we discuss the representation of the FECM and its relationship with the FAVAR. In Section 3 we derive the moving-average representation of the FECM and discuss structural identification schemes. In Section 4 we deal with estimation. In Section 5 we present the results of the Monte Carlo experiment. In Section 6 we discuss the two empirical applications. Finally, in Section 7 we summarize the main results and conclude. Appendices A and B present, respectively, an analytical example comparing the FAVAR and FECM responses and a comparison of the empirical FECM and DSGE based responses.

2 The Factor-augmented Error-Correction Model (FECM)

Consider the following dynamic factor model (DFM) for the I(1) scalar process X_{it}:

$$\begin{aligned} X_{it} &= \sum_{j=0}^{p} \lambda_{ij} F_{t-j} + \sum_{l=0}^{m} \phi_{il} c_{t-l} + \varepsilon_{it} \\ &= \lambda_i(L) F_t + \phi_i(L) c_t + \varepsilon_{it}, \end{aligned} \tag{1}$$

where $i = 1, ..., N$, $t = 1, ..., T$, F_t is an r_1-dimensional vector of random walks, c_t is an r_2-dimensional vector of I(0) factors, $F_t = c_t = 0$ for $t < 0$, and ε_{it} is a zero-mean idiosyncratic component. Both F_t and c_t are latent, unobserved variables. $\lambda_i (L)$ and $\phi_i (L)$ are lag polynomials of finite orders p and m respectively.

The loadings λ_{ij} and ϕ_{ij} are either deterministic or stochastic and satisfy the following restrictions. For $\lambda_i = \lambda_i(1)$ and $\phi_i = \phi_i(1)$ we have $E \|\lambda_i\|^4 \leq M < \infty$, $E \|\phi_i\|^4 \leq M < \infty$, and $1/N \sum_{i=0}^{N} \lambda_i \lambda_i'$, $1/N \sum_{i=0}^{N} \phi_i \phi_i'$ converge in probability to positive definite matrices. Furthermore, we assume that $E (\lambda_{ij} \varepsilon_{is}) = E (\phi_{ij} \varepsilon_{is}) = 0$ for all i, j and s.

As in Bai (2004), the idiosyncratic components ε_{it} are allowed to be serially and weakly cross correlated:

$$\varepsilon_t = \Gamma(L) \varepsilon_{t-1} + v_t,$$

where $\varepsilon_t = [\varepsilon_{1t,...,} \varepsilon_{Nt}]'$, and the vector process $v_t = [v_{1t,...,} v_{Nt}]'$ is white noise.

To derive the FECM and discuss further assumptions upon the model that ensure consistent estimation of the model's components, it is convenient to write first the DFM in static form. To this end, we follow Bai (2004) and define

$$\tilde{\lambda}_{ik} = \lambda_{ik} + \lambda_{ik+1} + ... + \lambda_{ip},\ k = 0, ..., p.$$

Let us in addition define

$$\tilde{\Phi}_i = [\phi_{i0}, ..., \phi_{im}]\,.$$

We can then obtain a static representation of the DFM which isolates the I(1) factors from the I(0) factors:

$$X_{it} = \Lambda_i F_t + \Phi_i G_t + \varepsilon_{it}, \tag{2}$$

where

$$\begin{aligned}
\Lambda_i &= \tilde{\lambda}_{i0}, \\
\Phi_i &= \left[\tilde{\Phi}_i, -\tilde{\lambda}_{i1}, \ldots, -\tilde{\lambda}_{ip}\right], \\
G_t &= \left[c_t', c_{t-1}', ..., c_{t-m}', \Delta F_t', ..., \Delta F_{t-p+1}'\right]'.
\end{aligned}$$

Introducing for convenience the notation $\Psi_i = [\Lambda_i', \Phi_i']'$, the following assumptions are needed for consistent estimation of both the I(1) and I(0) factors: $E\left\|\Psi_i\right\|^4 \leq M < \infty$ and $1/N\sum_{i=0}^{N}\Psi_i\Psi_i'$ converges to a $(r_1(p+1)+r_2(m+1)) \times (r_1(p+1)+r_2(m+1))$ positive-definite matrix.

Grouping across the N variables we have

$$X_t = \Lambda F_t + \Phi G_t + \varepsilon_t \tag{3}$$

where $X_t = [X_{1t}, ..., X_{Nt}]', \Lambda = [\Lambda_1^{'}, ..., \Lambda_N^{'}]', \Phi = [\Phi_1^{'}, ..., \Phi_N^{'}]'$ and $\varepsilon_t = [\varepsilon_{1t}, ..., \varepsilon_{Nt}]'$.

As noted above, the idiosyncratic component in (3) is serially correlated. This serial correlation can be eliminated from the error process by premultiplying (2) by

$$I - \Gamma\left(L\right)L$$

where

$$\Gamma\left(L\right) = \begin{bmatrix} \gamma_1\left(L\right) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \gamma_N\left(L\right) \end{bmatrix}.$$

Following this transformation, we obtain

$$X_t = \left(I - \Gamma\left(L\right)L\right)\Lambda F_t + \left(I - \Gamma\left(L\right)L\right)\Phi G_t + \Gamma\left(L\right)X_{t-1} + v_t.$$

Note that $\Gamma\left(L\right)$ can be conveniently factorized as

$$\Gamma(L) = \Gamma(1) - \Gamma_1(L)(1-L), \tag{4}$$

which allows us to rewrite the previous expression as

$$
\begin{aligned}
X_t = {} & \Lambda F_t + \Phi G_t - (\Gamma(1) - \Gamma_1(L)(1-L))(\Lambda F_{t-1} + \Phi G_{t-1}) \\
& + (\Gamma(1) - \Gamma_1(L)(1-L))X_{t-1} + v_t.
\end{aligned} \tag{5}
$$

With further manipulation we get

$$
\begin{aligned}
X_t = {} & \Lambda F_t + \Phi G_t - \Gamma(1)\Lambda F_{t-1} + \Gamma_1(L)\Lambda\Delta F_{t-1} - \Gamma(1)\Phi G_{t-1} \\
& + \Gamma_1(L)\Lambda\Phi\Delta G_{t-1} + \Gamma(1)X_{t-1} - \Gamma_1(L)\Delta X_{t-1} + v_t
\end{aligned} \tag{6}
$$

or

$$
\begin{aligned}
\Delta X_t = {} & \Lambda F_t + \Phi G_t - \Gamma(1)\Lambda F_{t-1} + \Gamma_1(L)\Lambda\Delta F_{t-1} - \Gamma(1)\Phi G_{t-1} \\
& + \Gamma_1(L)\Phi\Delta G_{t-1} - (I - \Gamma(1))X_{t-1} - \Gamma_1(L)\Delta X_{t-1} + v_t
\end{aligned} \tag{7}
$$

The ECM form of the DFM, i.e., the factor-augmented error-correction model (FECM), then follows directly as

$$
\begin{aligned}
\Delta X_t = {} & \underbrace{-(I - \Gamma(1))(X_{t-1} - \Lambda F_{t-1})}_{\text{Omitted in the FAVAR}} + \Lambda\Delta F_t + \Gamma_1(L)\Lambda\Delta F_{t-1} \\
& + \Phi G_t - \Gamma(1)\Phi G_{t-1} + \Gamma_1(L)\Phi\Delta G_{t-1} - \Gamma_1(L)\Delta X_{t-1} + v_t.
\end{aligned} \tag{8}
$$

Equation (8) is a representation of the DFM in (1) in terms of stationary variables. It contains the error-correction term, $-(I - \Gamma(1))(X_{t-1} - \Lambda F_{t-1})$, which is omitted in the standard FAVAR model that therefore suffers from an omitted variable problem, similar to the case of a VAR in differences in the presence of cointegration.

Note that it follows from (3) that

$$
X_{t-1} - \Lambda F_{t-1} = \Phi G_{t-1} + \varepsilon_{t-1}, \tag{9}
$$

such that it would appear at first sight that the omitted error-correction terms in the FAVAR could be approximated by including additional lags of the I(0) factors. However, by substituting the previous expression into (8) and simplifying we get

$$
\Delta X_t = \Lambda\Delta F_t + \Phi\Delta G_t + \Delta\varepsilon_t, \tag{10}
$$

which contains a non-invertible MA component. This is problematic from two points of view. Firstly, the structural identification schemes analyzed by Stock and Watson (2005) (see also the survey in Lütkepohl, 2014) rely on inverting the MA process in the idiosyncratic component, and on estimation of v_{it}, the i.i.d. part of the idiosyncratic component. In (10) the presence of a non-invertible MA process implies that v_{it} cannot be identified since the omission of the error-correction terms cannot be approximated with a finite number of lags of the I(0) factors (see on-line Appendix A). Secondly, even if the identification of structural shocks is based only on innovations to the factors and does not require estimation of v_{it}, as in Bernanke et al. (2005), inversion of the MA component is needed to get the endogenous lags in the equations for ΔX_{it}. These capture the variable-specific autoregressive dynamics that are unrelated to the common factors but affects the impulse responses of ΔX_{ti}.

To elaborate this point further consider the following example. Representation (10) can be alternatively written as

$$\Delta X_t = \Lambda \Delta F_t + \Phi(G_t - G_{t-1}) + \varepsilon_t - \varepsilon_{t-1},$$

which, by using (9) becomes

$$\Delta X_t = -(X_{t-1} - \Lambda F_{t-1}) + \Lambda \Delta F_t + \Phi G_t + \varepsilon_t. \tag{11}$$

At first sight, this is a model that contains an error-correction term, but has a much simpler structure than the FECM in (8). If the identification of structural shocks are based on innovations to dynamic factors, then such a model would appear to account for the omitted error-correction term in the FAVAR. Note, however, that in order to compute consistent impulse responses to innovations either to F_t or G_t, one still needs to invert the process ε_t so as to get the variable-specific autoregressive dynamics. By doing so, one obtains the FECM representation (8).

In sum, whenever we deal with I(1) data, and many macroeconomic series exhibit this feature, the standard FAVAR model potentially produces biased impulse responses unless we use an infinite number of factors as regressors, or account explicitly for the non-invertible MA structure of the error-process. The analytical example in on-line Appendix A elaborates this point further, and our simulation and empirical analyses below confirm that the omission of the ECM term in the FAVAR may potentially have an important impact on the impulse response functions obtained in typical macroeconomic applications.

To complete the model, we assume that the nonstationary factors follow a vector random walk process

$$F_t = F_{t-1} + \varepsilon_t^F, \tag{12}$$

while the stationary factors are represented by

$$c_t = \rho c_{t-1} + \varepsilon_t^c, \tag{13}$$

where ρ is a diagonal matrix with values on the diagonal in absolute term strictly less than one. ε_t^F and ε_t^c are independent of λ_{ij}, ϕ_{ij} and ε_{it} for any i, j, t. As in Bai (2004), it should be noted that the error processes ε_t^F and ε_t^c need not necessarily be *i.i.d.*. They are allowed to be serially and cross correlated and jointly follow a stable vector process:

$$\begin{bmatrix} \varepsilon_t^F \\ \varepsilon_t^c \end{bmatrix} = A(L) \begin{bmatrix} \varepsilon_{t-1}^F \\ \varepsilon_{t-1}^c \end{bmatrix} + \begin{bmatrix} u_t \\ w_t \end{bmatrix}, \tag{14}$$

where u_t and w_t are zero-mean white-noise innovations to dynamic nonstationary and stationary factors, respectively. Under the stability assumption, we can express the model as

$$\begin{bmatrix} \varepsilon_t^F \\ \varepsilon_t^c \end{bmatrix} = [I - A(L)L]^{-1} \begin{bmatrix} u_t \\ w_t \end{bmatrix}. \tag{15}$$

Using (12), (13) and (15) we can write the VAR for the factors as

$$\begin{aligned} \begin{bmatrix} F_t \\ c_t \end{bmatrix} &= \left[\begin{bmatrix} I & 0 \\ 0 & \rho \end{bmatrix} + A(L) \right] \begin{bmatrix} F_{t-1} \\ c_{t-1} \end{bmatrix} - A(L) \begin{bmatrix} I & 0 \\ 0 & \rho \end{bmatrix} \begin{bmatrix} F_{t-2} \\ c_{t-2} \end{bmatrix} + \begin{bmatrix} u_t \\ w_t \end{bmatrix} \\ &= C(L) \begin{bmatrix} F_{t-1} \\ c_{t-1} \end{bmatrix} + \begin{bmatrix} u_t \\ w_t \end{bmatrix}, \end{aligned} \tag{16}$$

where the parameter restrictions imply that $C(1)$ is a block-diagonal matrix with block sizes corresponding to the partition between F_t and c_t.

The FECM is specified in terms of static factors F and G, which calls for a corresponding VAR specification. Using the definition of G_t and (16) it is straightforward to get the following representation

$$
\begin{bmatrix}
I & 0 & \dots & & & \dots & 0 \\
0 & I & \dots & & & \dots & 0 \\
\vdots & & & & & & \vdots \\
\vdots & & & & & & \vdots \\
0 & \dots & I & 0 & & \dots & 0 \\
-I & \dots & 0 & I & 0 & \dots & 0 \\
0 & \dots & 0 & 0 & I & \dots & 0 \\
\vdots & & & & & & \vdots \\
0 & \dots & & \dots & & \dots & I
\end{bmatrix}
\begin{bmatrix}
F_t \\ c_t \\ c_{t-1} \\ \vdots \\ c_{t-m} \\ \Delta F_t \\ \Delta F_{t-1} \\ \vdots \\ \Delta F_{t-p+1}
\end{bmatrix} =
$$

$$
\begin{bmatrix}
C_{11}(L) & C_{12}(L) & 0 & \dots & & & \dots & 0 \\
C_{21}(L) & C_{22}(L) & 0 & \dots & & & \dots & 0 \\
0 & I & 0 & \dots & & & \dots & 0 \\
\vdots & & \dots & & \dots & & & \vdots \\
0 & & \dots & \dots & I & 0 & \dots & 0 \\
-I & & \dots & & \dots & & \dots & 0 \\
0 & & \dots & & & I & \dots & 0 \\
\vdots & & & & & & & \vdots \\
0 & & \dots & & \dots & & I & 0
\end{bmatrix}
\begin{bmatrix}
F_{t-1} \\ c_{t-1} \\ c_{t-2} \\ \vdots \\ c_{t-m-1} \\ \Delta F_{t-1} \\ \Delta F_{t-2} \\ \vdots \\ \Delta F_{t-p}
\end{bmatrix} +
\begin{bmatrix}
I & 0 \\ 0 & I \\ 0 & 0 \\ & \\ \vdots & \vdots \\ & \\ \vdots & \vdots \\ & \\ 0 & 0
\end{bmatrix}
\begin{bmatrix}
u_t \\ w_t
\end{bmatrix} \qquad (17)
$$

Using the definition of G_t, the VAR for the static factors, and premultplying the whole expression by the inverse of the initial matrix in (17), the factor VAR can be more compactly written as

$$
\begin{bmatrix} F_t \\ G_t \end{bmatrix} = \begin{bmatrix} M_{11}(L) & M_{12}(L) \\ M_{21}(L) & M_{22}(L) \end{bmatrix} \begin{bmatrix} F_{t-1} \\ G_{t-1} \end{bmatrix} + Q \begin{bmatrix} u_t \\ w_t \end{bmatrix}, \qquad (18)
$$

where the $(r_1(p+1) + r_2(m+1)) \times (r_1 + r_2)$ matrix Q accounts for dynamic singularity of G_t. This is due to the fact that the dimension of the vector process w_t is r_2, which is smaller than or equal to $q = r_1 p + r_2(m+1)$, the dimension of G_t. Let us assume that the order of the VAR in (18) is n.

We conclude this section with a few remarks on the generality and empirical relevance of our framework. First, the model contains both I(1) factors - common trends, and I(0) factors - common cycles. As we show in our empirical examples (see Section 4.2), the presence of I(0) factors in non-stationary data is well supported by the data, making our formulation more relevant to the identification, estimation and structural analysis undertaken in the paper. Moreover, the idiosyncratic errors can also be either I(0) or I(1).

Second, Bai (2004) shows that the model accommodates potential cointegration among non-stationary factors F_t. If F_t are cointegrated, then there exists a full-rank matrik P

such that $PF_t = [\xi_t', \eta_t']'$, where ξ_t is a vector of non-cointegrated I(1) factors, while η_t are I(0) linear combinations of F_t. One can then simply redefine ξ_t as F_t and include η_t into G_t. In other words, model (1) and a model with explicitly cointegrated F_t are observationally equivalent and there is no loss of information when moving from the latter to the former as long as the space spanned by the I(1) and I(0) factors can be consistently estimated. This fact is very convenient for our analysis. Namely, the focus of our analysis is on impulse responses of observable variables to structural innovations to factors, where the structural innovations are identified by imposing restrictions on the effects of factors on observable variables X_t. Model (1) in such a case produces consistent responses to structural shocks, allowing us to focus on the effect of error-correction mechanism in observable variables as incorporated in the FECM (8) on impulse responses to identified structural shocks.

Third, Barigozzi et al. (2016a, 2016b) focus on the case of cointegrated factors and develop a detailed and interesting representation and estimation theory based on more primitive assumptions that those in Bai (2004) and Bai and Ng (2004). Instead, as mentioned, we simply rely on Bai (2004) and Bai and Ng (2004), and focus more on evaluating the practical relevance for applied econometrics of accounting for cointegration in a large factor model context.

Finally, by comparing the dynamic factor model (1) to the static factor representation (3) one observes that stochastic singularity of the factor space is confined to the subspace spanned by the stationary factors.

3 Moving-average representation of the FECM and the Structural FECM

The identification of structural shocks in VAR models usually rests on imposing restrictions upon the parameters of the moving-average representation of the VAR. An analogous approach is used in the FAVAR model, where the moving-average representation is derived jointly for the observable variables and the factors (Stock and Watson, 2005; Lütkepohl, 2014). In the case of I(1) and cointegrated variables it is useful to work with the VAR in vector-error correction form as the Wold moving-average representation will not exist in general for the VAR in levels (Lütkepohl, 2005). In this respect, Johansen (1995, Theorem 4.2) proves that the moving-average representation of the VECM exists and its structure can be conveniently exploited for the development of identification schemes based on long-run restrictions (King et al. 1991; Lütkepohl, 2005). The motivation for Johansen's approach to the derivation of the Granger representation theorem is straightforward. The VAR model, and not the moving-average model, is usually fitted to the data. Under cointegration, the VAR can be conveniently reparameterized as a VECM and the moving-average form obtained.

The FECM is a generalization of error-correction models to large dynamic panels. In the derivation of the corresponding moving-average representation we follow a similar logic as above. A VAR is commonly fitted to the estimated factors in the FAVAR analysis and it thus presents a natural starting point for the analysis. With I(1) factors it is convenient to work with the corresponding error-correction form. Because of the importance of the error-correction mechanism for the observation equations, we argue that the FECM equations (8) are best fitted to the data. By combining the VECM representation of VAR (for the factors) and the FECM equations we derive the moving-average representation of the FECM, which represents a generalization of the Johansen (1995) version of the Granger representation theorem to cointegrated non-stationary panels. Based on it, we then discuss shock identification.

3.1 The MA representation of the FECM

To start with, we conveniently reparameterize the factor VAR process (18). It contains r_1 unit roots pertaining to F_t, while G_t is an I(0) process. In the exposition of the model it is useful to consider the possibility that only the space jointly spanned by F_t and G_t can be estimated, and not the corresponding spaces of F_t and G_t separately. On the other hand, since our interest is in identifying structural shocks, estimation of F_t and G_t separately is not even necessary. To this end, we introduce the following joint vector of factors $\tilde{F}_t = [F_t', G_t']'$ of dimension $r = r_1 + q$, while the corresponding factors loadings are stacked as $\tilde{\Lambda} = [\Lambda', \Phi']'$. With this notation (18) can be written as

$$\tilde{F}_t = M(L)\tilde{F}_{t-1} + \tilde{u}_t \tag{19}$$

where $\tilde{u}_t = Q[u_t', w_t']'$. Because the matrix $[I - \sum_i M_i(1)]$ has rank $q = r - r_1$ we can rewrite (19) in error-correction form

$$\Delta\tilde{F}_t = \alpha_F \beta_F' \tilde{F}_{t-1} + M^*(L)\Delta\tilde{F}_{t-1} + \tilde{u}_t \tag{20}$$

where the coefficient matrices of the matrix polynomials $M_{ij}^*(L)$ are defined from the coefficient matrices in (18) as:

$$M_{ijl}^* = -(M_{ijl+1} + ... + M_{ijn}), \quad l = 1, \ldots, n-1. \tag{21}$$

and α_F and β_F are full rank $r \times q$ matrices respectively. For future reference let $\alpha_{F,\perp}$ and $\beta_{F,\perp}$ be their corresponding orthogonal complements, which can be determined as $\alpha_\perp = [I_q - c(\beta_F'^{-1}\beta']c_\perp$, where $c = [I_q, 0]'$ and $c_\perp = [0, I_{r_1}]'$, and similarly for $\beta_{F,\perp}$. Estimation of parameters of (20) is discussed below. At this point we can state the following proposition.

Proposition 1 (Moving-average representation of the FECM) *Given the error-correction representations of the dynamic factor model (8) and (20), the moving-average representa-*

tion of the factor-augmented error-correction model is

$$\begin{bmatrix} X_t \\ \tilde{F}_t \end{bmatrix} = \begin{bmatrix} \tilde{\Lambda} \\ I_r \end{bmatrix} C_F \sum_{i=1}^{t} \tilde{u}_i + C_1(L) \begin{bmatrix} v_t + \tilde{\Lambda}\tilde{u}_t \\ \tilde{w}_t \end{bmatrix}. \tag{22}$$

where $C_F = \beta_{F,\perp}\omega\alpha'_{F,\perp}$ *and*

$$\omega = \left(\alpha'_{\perp}\Xi\beta_{\perp}\right)^{-1} = \left[\alpha'_{F,\perp}(I_r - M^*(L))\beta_{F,\perp}\right]^{-1}.$$

A necessary and sufficient condition for the existence of this representation is $|\omega| \neq 0$.

Proof. The FECM (8) can be rewritten as

$$\Delta X_t = \tilde{\alpha}(X_{t-1} - \tilde{\Lambda}\tilde{F}_{t-1}) + \tilde{\Lambda}\Delta\tilde{F}_t + \Gamma_1(L)\tilde{\Lambda}\Delta\tilde{F}_{t-1} - \Gamma_1(L)\Delta X_{t-1} + v_t, \tag{23}$$

where $\tilde{\alpha} = -(I - \Gamma(1))$. Using (20) we can stack the equations for ΔX_t and the factors into a single system of equations as

$$\begin{bmatrix} \Delta X_t \\ \Delta\tilde{F}_t \end{bmatrix} = \alpha\beta' \begin{bmatrix} X_{t-1} \\ \tilde{F}_{t-1} \end{bmatrix} + \begin{bmatrix} -\Gamma_1(L) & B(L) \\ 0 & M^*(L) \end{bmatrix} \begin{bmatrix} \Delta X_{t-1} \\ \Delta\tilde{F}_{t-1} \end{bmatrix} + \begin{bmatrix} v_t + \tilde{\Lambda}\tilde{u}_t \\ \tilde{w}_t \end{bmatrix} \tag{24}$$

where $B(L) = \tilde{\Lambda}M^*(L) + \Gamma_1(L)\tilde{\Lambda}$ and

$$\underset{(N+r)\times(N+q)}{\alpha} = \begin{bmatrix} \tilde{\alpha} & \tilde{\Lambda}\alpha_F \\ 0 & \alpha_F \end{bmatrix} \text{ and } \underset{(N+q)\times(N+r)}{\beta'} = \begin{bmatrix} I & -\tilde{\Lambda} \\ 0 & \beta_F \end{bmatrix}.$$

We can observe that (24) has a structure similar to a standard ECM model with some restrictions imposed. There are $N + r_1 + q$ variables driven by r_1 common stochastic trends and therefore there are $N + q$ cointegration relations. The model conforms with the assumptions of the Johansen's version of the Granger representation theorem (Johansen, 1995, Theorem 4.2). In particular

$$\beta_{\perp} = \begin{bmatrix} \tilde{\Lambda} \\ I_r \end{bmatrix} \beta_{F,\perp},\ \alpha_{\perp} = \begin{bmatrix} 0_{N\times r_1} \\ \alpha_{F,\perp} \end{bmatrix},\ \Xi = I_{N+r} - \begin{bmatrix} -\Gamma_1(1) & B(1) \\ 0 & M^*(1) \end{bmatrix}$$

and

$$\omega = \left(\alpha'_{\perp}\Xi\beta_{\perp}\right)^{-1} = \left[\alpha'_{F,\perp}(I_r - M^*(L))\beta_{F,\perp}\right]^{-1}$$

is a full rank matrix by the assumption that the data are at most I(1).[2] Then the generic moving-average representation by the Granger representation theorem can be written as[3]

$$\begin{bmatrix} X_t \\ \tilde{F}_t \end{bmatrix} = C \sum_{i=1}^{t} \tilde{u}_i + C_1(L) \begin{bmatrix} v_t + \tilde{\Lambda}\tilde{u}_t \\ \tilde{u}_t \end{bmatrix},$$

[2] X_{it} are assumed to be at most I(1). If the data were I(2) processes, ω would be singular.

[3] The result obtains from replication of the proof of Theorem 4.2 in Johansen (1995).

with

$$C = \beta_\perp \left(\alpha'_\perp \Xi \beta_\perp\right)^{-1},$$

which simplifies to (22). ■

The derivation of (22) is also instructive to compare our modelling framework to that of Barigozzi et al. (2016a). Their starting point is the moving-average representation for $\tilde{F}_t$ from which they derive (20) by replicating the proof of Theorem 4.5 in Johansen (1995). Our starting point is instead the VAR representation for $\tilde{F}_t$ (19), which can be reparameterized as the VECM representation of the common components (20). We combine it with the FECM representation for observable variables (23) to obtain a moving-average representation for observable variables, as in Johansen's Theorem 4.2. Therefore, we focus on studying the effects of the error-correction mechanism on the observable variables, which is particularly convenient to develop an identification scheme for structural shocks based on the long-run effects of innovations to dynamic factors on observable variables.

3.2 Structural FECM

Our model contains I(1) and I(0) factors with corresponding dynamic factors innovations. From the moving-average representation (22) we can observe that the innovations in the first group have permanent effects on X_t, while the innovations in the second group have only transitory effects. The identification of structural dynamic factor innovations can be performed separately for each group of structural innovations or on both simultaneously. As is standard in SVAR analysis, we assume that structural dynamic factor innovations are linearly related to the reduced-form innovations

$$\varphi_t = \begin{bmatrix} \eta_t \\ \mu_t \end{bmatrix} = H \begin{bmatrix} u_t \\ w_t \end{bmatrix}, \tag{25}$$

where H is a full-rank $(r_1 + r_2) \times (r_1 + r_2)$ matrix. η_t are r_1 permanent structural dynamic factor innovations and μ_t are r_2 transitory structural dynamic factor innovations. It is assumed that $E\varphi_t\varphi_t' = I_{r_1+r_2}$ such that $H\Sigma_{u,w}H' = I_{r_1+r_2}$.

The moving average representation of the FECM in structural form can be obtained by inserting the two linear transformations above of reduced-form innovations to dynamic factors into the moving-average representation of the FECM given by (22).

3.3 Identifying structural shocks through long-run restrictions

The three most common classes of identification restrictions in the SVAR literature are contemporaneous restrictions, long-run restrictions and sign restrictions. Even though we provide also an example of identification of monetary policy shocks through contemporaneous restrictions, the focus of this paper is on long-run restrictions. Specifically, we extend the analysis of structural common stochastic trends of King et al. (1991) to the case of large nonstationary panels.[4]

The identification of structural innovations by means of long-run restrictions can be obtained by imposing constraints on the matrices Λ and ω in the moving-average representation of the FECM (22). By doing this, we replace the long-run effects of reduced-form innovations to factors u_t, i.e.,

$$\Lambda C_F \sum_{i=1}^{t} u_i,$$

with the long-run effects of structural innovations denoted η_t, i.e.,

$$\Lambda^* C_F^* \sum_{i=1}^{t} \eta_i,$$

where the matrices Λ^* and C_F^* contain restrictions motivated by economic theory.

A common economically motivated identification scheme of permanent shocks, originally proposed by Blanchard and Quah (1989), uses the concept of long-run money neutrality. In this respect, their identification scheme distinguishes real from nominal shocks by imposing zero long-run effects of the nominal shock on real variables.

In a cointegration framework such an identification approach was formalized by King et al. (1991) (see also Warne, 1993). King et al. (1991) analyzed a six-dimensional system of cointegrated real and nominal variables. By imposing a particular cointegration rank, they determined the subset of innovations with permanent effects. Within this subset, they restricted the number of real stochastic trends to one, and identified it by imposing zero restrictions on real variables of all other permanent shocks in the subset. The remaining permanent shocks were allowed to have non-zero effects only on the subset of nominal variables in the cointegrated VAR. We extend the identification approach of King et al. (1991) to large-dimensional panels of non-stationary data using the FECM.

The FECM contains r_1 stochastic trends. Consider the case where $r_1 = 2$. We have two I(1) factors and want to identify the innovations to the first as real permanent shocks and innovations to the second as nominal permanent shocks. Accordingly, partition the variables in X_t such that N_1 real variables are ordered first and the remaining $N_2 = N - N_1$ nominal variables are ordered last. The group of real variables contains various measures of economic activity measured in levels, e.g. indexes of industrial production, which are treated as I(1). The identifying restrictions would thus be that the innovations to the nominal stochastic trend have a zero long-run effect on these variables. Since nominal variables, for example, the levels of different price indexes and nominal wages, are grouped at the bottom of the panel, the restricted loading matrix Λ^* would have the following

[4]Barigozzi et al. (2016b) also provide an empirical example of a long-run identification scheme in a large-scale modelling framework. In their empirical application they impose long-run restrictions on two real variables only (GDP and investment), so that some nominal shocks could still have a non-zero long-run effect on some of the other observable real variables. Instead, our identification scheme imposes zero-long run effects of nominal shocks on all observable real variables in the data.

structure:

$$\Lambda^* = \begin{bmatrix} \Lambda_{11}^* & \mathbf{0} \\ \Lambda_{21}^* & \Lambda_{22}^* \end{bmatrix},$$

where Λ_{11}^* is $N_1 \times 1$ and Λ_{21}^* and Λ_{22}^* are $N_2 \times 1$. More generally, if the objective were to identify only the real permanent shocks with $r_1 > 2$, the dimension of Λ_{22}^* would be $N_2 \times (r_1 - 1)$.

The matrix Λ^* can be identified in the following way. First, the real stochastic trend is allowed to load on all observable variables. This implies that Λ_{11}^* and Λ_{21}^* can be identified as loadings to the first factor - F_t^r - extracted from the whole dataset. Second, we can estimate the residuals from a projection of X_t on F_t^r. Denote these as ε_t^r. Then Λ_{22}^* is identified as the loadings to the $(r_1 - 1)$ factors - denoted F_t^n - extracted from the lower N_2-dimensional block of ε_t^r.

Note that block diagonality of Λ^* alone does not ensure that nominal shocks do not affect real variables, but we also need to impose the corresponding zero restrictions on C_F^* since it is the product $\Lambda^* C_F^*$ that determines the long-run effects of structural factor innovations to observable variables. This implies that zero long-run effect restrictions require $\Lambda^* C_F^*$ to be lower block diagonal, which is achieved by imposing restrictions on C_F^* in addition to lower (block) diagonality of Λ^*.

Restrictions on C_F^* are imposed in the following way (see Lütkepohl (2005) for details). There are q stationary factors in the system. Their corresponding innovations have zero long-run effects. This implies that C_F^* has q columns of zeros. On the parameters of the remaining r_1 columns we need to impose $r_1(r_1-1)/2$ zero restrictions. Just identification of structural innovations to factors is obtained by imposing additional $q(q-1)/2$ restrictions on H. For a discussion of estimation of C_F^* under such restrictions see Section 4.2.

In sum, the identification scheme of structural stochastic shocks assumes that factors are unconditionally uncorrelated, which combined with the imposed structure on Λ^* and C_F^* allows us to identify the innovations to the first estimated factor as the permanent real (productivity) shocks. Note also that such an identification scheme is based on long-run effects of innovations to dynamic factors on observable variables, which means that it is essential to derive it from the moving-average representation of the FECM in (22). Given that economic theory motivates the use of identifying restrictions in relation to observables this is a practical advantage of our approach with respect to other applications where restrictions are specified in terms of latent variables, as e.g. in Barigozzi et al. (2016b).

4 Estimation of the FECM

In the dynamic factor model, the idiosyncratic components ε_{it} are allowed to be serially and weakly cross correlated as in Bai (2004) and Bai and Ng (2004). Specifically, along the time series dimension, $\varepsilon_{it} = \gamma_i(L)\varepsilon_{it-1} + v_{it}$. If $\gamma_i(L)$ contains a unit root for some i, for those i, X_{it} and F_t do not cointegrate. However, the factorization of $\Gamma(L)$ in (4) remains valid, so that the derivation of the FECM does not need the assumption of stationary idiosyncratic components. Yet, allowing for the possibility of I(1) idiosyncratic errors requires some modifications in the estimation procedure. Therefore, we now discuss FECM estimation first assuming all idiosyncratic errors are I(0), and then relax this assumption to allow for some I(1) errors.

4.1 Estimation of the FECM with stationary idiosyncratic components

There are several reasons for making the hypothesis of I(0) idiosyncratic errors. From an economic theory point of view, integrated errors are unlikely as they would imply that the integrated variables can drift apart in the long run, contrary to general equilibrium arguments. Empirically, integrated variables that drift apart are likely to be few and of marginal importance.[5]

Stationarity of ε_{it} implies that X_{it} and F_t cointegrate for all i. Naturally, this does not imply that all bivariate pairs of variables X_{it} and X_{jt}, $j \neq i$, cointegrate mutually. In fact, when the idiosyncratic errors are I(0), if there are N variables and r I(1) common factors, $r \leq N-1$, then all subsets of $r+1$ variables are cointegrated.

With stationary idiosyncratic components the FECM model is consistent with the specification of the dynamic factor model analyzed by Bai (2004), which accommodates the presence of I(0) factors along with I(1) factors. Our assumptions are consistent with Bai's (2004) and we can therefore rely on Bai's (2004) results on the asymptotic properties of the principal component based factor (and loadings) estimators.

Specifically, the space spanned by the factors can be consistently estimated using principal components. The estimators of the space spanned by F_t, denoted $\widetilde{F}$, are the eigenvectors corresponding to the largest r_1 eigenvalues of XX' normalized such that $\widetilde{F}'\widetilde{F}/T^2 = I$. The space spanned by the stationary factors G_t, denoted $\widetilde{G}$, can be estimated as the eigenvectors corresponding to the next q largest eigenvalues normalized such that $\widetilde{G}'\widetilde{G}/T = I$ (Bai, 2004). Corresponding estimators of the loadings to I(1) factors are then $\widetilde{\Lambda} = X'\widetilde{F}/T^2$, and those to the I(0) factors $\widetilde{\Phi} = X'\widetilde{G}/T$.[6] Moreover, with spaces spanned by F_t and G_t estimated separately, we have $E\left\|\varepsilon_t^F\right\|^4 \leq M < \infty$, which implies that $1/T^2\sum_{t=1}^T F_tF_t'$, $1/T\sum_{t=1}^T G_tG_t'$, and the cross-product matrices $1/T^{3/2}\sum_{t=1}^T F_tG_t'$ and $1/T^{3/2}\sum_{t=1}^T G_t'F_t$ converge. The elements of the matrix composed of these four elements

[5] In the empirical application discussed in Section 6 we use a monthly US dataset for the period 1959 - 2014 (McCracken and Ng, 2015). The dataset contains 128 series, of which 114 are treated as non-stationary. When applying the ADF unit root test to the estimated idiosyncratic components, the unit-root null is not rejected at the 5% significance level for only 8 out of the 114 series, and for only 4 of them at the 10% level. Simulation evidence in Westerlund and Larsson (2009) shows that for $T >> N$, which is the case of our empirical application, ADF tests on estimated idiosyncratic components behaves well, but can in many other cases behave poorly in terms of size and power. For this reason we discuss the estimation of the FECM under both specifications of the order of integration of ε_{it}. Moreover, the estimators used in our empirical application allow for I(1) idiosyncratic components.

[6] In a model similar to ours, Choi (2011) analyzes the generalized principal components estimator that offers some efficiency gains over the classic principal components estimator. Simulation evidence presented below, however, shows that Bai's estimator performs very well even with small sample sizes. For this reason we stick to the standard principal components estimator in this paper.

jointly converge to form a positive definite matrix.

Using the estimated factors and loadings, the estimates of the common components are $\widetilde{\Lambda}\widetilde{F}_t$, $\widetilde{\Phi}\widetilde{G}_t$, $\widetilde{\Lambda}\Delta\widetilde{F}_t$ and $\widetilde{\Phi}\Delta\widetilde{G}_t$, while for the cointegration relations it is $X_{t-1} - \widetilde{\Lambda}\widetilde{F}_{t-1}$. Replacing the true factors and their loadings with their estimated counterparts is permitted as long as N is large relative to T and the assumptions discussed above and those in Bai (2004) (see Bai (2004) Lemma 2) are valid, so that there is no generated regressor problem.[7]

The estimated common components and cointegrating relations can be then used in (8) to estimate the remaining parameters of the FECM equation by equation by OLS. Banerjee, Marcellino and Masten (2014b) demonstrate by means of a Monte Carlo experiment that such an approach has very good finite sample properties both in terms of estimation of the factors space and impulse responses to structural shocks.

Finally, the number of I(1) factors r_1 can be consistently estimated using the criteria developed by Bai (2004) applied to data in levels. The overall number of static factors $r_1(p+1) + r_2(m+1)$ can be estimated using the criteria of Bai and Ng (2002) applied to the data in differences.

It is worth noting that the fact that we can separately estimate the spaced spanned by F_t and G_t under the I(0) ε_{it} assumption simplifies the structure and estimation of the FECM. In particular, we have

$$\alpha_F = \begin{bmatrix} 0 \\ \alpha_M \end{bmatrix}, \quad \beta_F' = \begin{bmatrix} 0 & I_{r_2} \end{bmatrix}, \quad C_F = \left[\left(I_{r_1} - \widehat{M}_{11}(1)\right)\right]^{-1}.$$

4.2 Estimation of the FECM with some I(1) idiosyncratic components

While I(1) idiosyncratic errors are unlikely, their presence cannot be a priori ruled out. Therefore, it is convenient to have a more general estimation method that allows for them. Luckily, our framework easily accommodates such a feature.

With some of the ε_{it} being I(1) the $(I - \Gamma(1))$ matrix in (8) contains rows of zeros for all those variables with I(1) idiosyncratic components. The result is expected. A non-stationary idiosyncratic component implies no cointegration between the corresponding X_{it} and F_t. In such a case, there is also no corresponding error-correction mechanism in the equation for ΔX_{it} in (8) for those i whose $\gamma_i(L)$ contain a unit root.

Consistent estimation of the factor space and the corresponding loading matrices can proceed as in Bai and Ng (2004). Namely, the principal components based estimator is applied to data transformed to I(0), and then cumulated to obtain the estimate of the r-dimensional space spanned by F_t and G_t, where r can be determined by the above-mentioned criteria of Bai and Ng (2002). This procedure does not, however, deliver the estimates of the spaces spanned by F_t and G_t separately. Yet, the use of the FECM does

[7] These assumptions are essentially (1) the common factor structure of the data, (2) heterogeneous loadings with finite fourth moments, (3) mutual orthogonality between u_t, w_t, ε_{it}, λ_{it} and ϕ_{it},, (4) weak dependence of idiosyncratic errors, and (5) N large compared with T for the $I(0)$ factors ($\sqrt{T}/N \to 0$).

not rely on separate estimation of the spaces spanned by F_t and G_t. It is definitely not needed either for forecasting or structural analysis. For the identification of structural shocks based on long-run restrictions we only need the estimates of some rotation of α_F and β_F in (20). In empirical applications below we choose to set the upper $q \times q$ block of β_F to the identity matrix I_q.

Moreover, estimation and testing in the model (20) can be based on the methodology developed by Johansen (1995). This implies that we can apply the Johansen trace test to determine the cointegration rank q, and then the number of I(1) factors is $r_1 = r - q$. Alternatively, r_1 can be determined directly by the MQ statistics proposed by Bai and Ng (2004). In this case, the parameters of (20) can be estimated by reduced-rank regression (Anderson, 1951).

The matrix Q that accounts for the dynamic singularity of innovations to static factors can in principle be estimated along the lines proposed in Stock and Watson (2005). After normalizing the variance-covariance matrix of $\tilde{u}_t$ to an identity matrix, they estimate Q as the sample eigenvectors of the long-run variance-covariance matrix of X_t accounted for by the common components $\tilde{\Lambda}\tilde{F}_t$. For the FECM such a procedure is easy to apply. Given the moving-average representation of the FECM (22) the estimator of Q would be the largest r_1+r_2 eigenvectors of the sample estimates of the $(r_1+r_2)\times(r_1+r_2)$ matrix $C_F'\tilde{\Lambda}'\tilde{\Lambda}C_F$. To apply the estimator we can determine the total number of innovations to dynamic factors $r_1 + r_2$ by the criteria proposed by Bai and Ng (2007) applied to the sample estimate of the variance-covariance matrix of innovations to the static factors. Alternatively, one can apply the Bai and Ng (2002) criteria the estimate of the variance-covariance matrix of v_t, as proposed by Stock and Watson (2005) and Amengual and Watson (2007).

As discussed in Section 3.3, identification of structural shocks involves imposing zero restrictions on C_F^* and H. The estimates of these matrices can then be obtained by maximum likelihood, as discussed in Vlaar (2004), see also Lütkepohl (2005).

The remaining parameters of the FECM can then be estimated as discussed above. The use of estimated factors rather than true factors does not create a generated regressor problem, as long as the longitudinal dimension grows faster than the temporal dimension; the precise condition is $T^{1/2}/N$ is $o(1)$, see Bai and Ng (2006).

5 An evaluation of the effects of the error-correction terms on impulse response analysis

In this section we analyze the effects of omitting the error-correction terms on impulse response analysis by means of simulation experiments, focusing on the role of the strength of error correction and of the sample size, along both the time series and cross section dimensions. In the design of the data-generating process we draw from the empirical analysis of real stochastic trends that is presented in detail in the next section. Given that the FECM and the FAVAR are set up such that the only difference between the two is the presence of the error-correction terms, the simulation evidence presented in this section also facilitates the discussion of the empirically observed differences.

The experiment is designed as follows. We estimate the FECM model (23) for the subset of I(1) variables in the US data panel and use the estimated parameters as DGP. The only exceptions are the loading coefficients of the cointegration relations, α. These are drawn from a uniform distribution around mean values $[-0.5, -0.25, -0.125]$ in order to assess the effects of error-correction strength. The idiosyncratic components of the data are treated as serially independent and randomly drawn with replacement from empirical residuals. The data are driven by factors simulated with the parameters from the estimated factor VAR, combined with randomly drawn factor VAR residuals.

Our panel contains 54% of real variables and 46% of nominal variables. This relative share is also preserved in the artificially generated data, i.e. out of N generated variables, 54% have parameters that are randomly drawn from the parameters pertaining to real variables. The rest are randomly drawn from the parameters of the subset of nominal variables.

We consider five different parameter configurations. The benchmark sample setup is with $T = 500$ and $N = 100$, which corresponds to the dataset from which the parameters used in the DGP are estimated. The benchmark mean value of the error-correction coefficient α is set to -0.50. We then vary the strength of error correction by setting the mean α to -0.25 and -0.125 respectively. The remaining two modifications alter the sample size. First, we halve the time series dimension to 250, and second we halve the cross-section dimension to 50. The number of Monte Carlo replications is set to 100.

Comparison between the FECM and the FAVAR impulse responses is done in three ways. First, we follow Olivei and Tenreyro (2010) who propose the use of the following two statistics[8]

$$D = \sup_{t\in[1,h]} |IR_{FECM,t} - IR_{FAVAR,t}|, \quad CD = |\sum_{t\in[1,h]} IR_{FECM,t} - IR_{FAVAR,t}|.$$

We set $h = 60$ and compute the bootstrapped p-values of D and CD statistics using 100 bootstrap replication within each Monte Carlo iteration. In the bootstrap procedure we first generate the factors by resampling the estimated innovations from the factor VAR (keeping the other coefficients fixed). Resampled factors are then used in the FAVAR equations with resampled idiosyncratic errors to generate the resampled observable variables that are used to estimate the FECM and the FAVAR parameters. The numer of factors and the structure of the factor-VAR model and FECM equations are kept fixed in the bootstrap procedure.

The second comparison is based on relative mean squared errors (MSE) of impulse

[8] We thank an anonymous referee for suggesting this test to us.

responses to a permanent productivity shock.[9] The third comparison is based on the share of FECM impulse responses outside the bootstrapped FAVAR confidence intervals.

Table 1: Importance of the error-correction term - results of the Monte Carlo experiment

Horizon	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
α	-0.50		-0.25		-0.125		-0.50		-0.50	
T	500		500		500		250		500	
N	100		100		100		100		50	
Panel A: **Olivei and Tenreyro test**										
Rejection frequency of equality of impulse responses										
Sig. level (%)	5	10	5	10	5	10	5	10	5	10
D statistic	99.92	99.96	99.94	99.97	99.97	99.98	99.93	99.98	99.94	99.98
CD statistic	99.81	99.86	99.80	99.90	99.67	99.76	99.54	99.68	99.76	99.82
Panel B:**Relative MSE - FECM/FAVAR**										
3	0.097		0.116		0.107		0.161		0.100	
6	0.047		0.066		0.074		0.085		0.048	
12	0.055		0.065		0.070		0.115		0.052	
18	0.073		0.076		0.079		0.165		0.068	
24	0.095		0.095		0.093		0.218		0.087	
36	0.148		0.142		0.134		0.328		0.135	
48	0.205		0.195		0.181		0.427		0.189	
60	0.256		0.243		0.225		0.507		0.239	
Panel C:**% of FECM responses outside the FAVAR conf. intervals**										
Confidence interval coverage (%)										
	67	90	67	90	67	90	67	90	67	90
3	78.14	56.93	77.69	56.31	71.53	46.67	72.83	50.27	82.6	66.56
6	94.05	89.08	92.73	86.18	90.91	83.64	90.90	84.85	94.8	91.62
12	97.62	95.66	96.83	94.7	96.46	93.89	96.29	93.68	98.24	96.70
18	98.27	96.98	98.23	96.75	97.52	95.83	97.83	96.23	98.76	97.62
24	98.85	97.99	98.61	97.58	98.05	96.72	98.2	96.73	99.04	98.38
36	99.10	98.29	99.09	98.23	98.53	97.35	98.73	97.85	99.36	98.72
48	99.22	98.53	99.00	98.28	98.97	97.96	98.89	98.23	99.52	99.04
60	99.28	98.84	99.21	98.71	99.02	98.18	99.08	98.45	99.48	99.04

The results of the Monte Carlo experiment are presented in Table 1. The Olivei and Tenreyro (2010) tests are reported in Panel A, which shows the rejection frequencies of the hypothesis that the FECM and the FAVAR produce the same impulse responses at bootstrapped 5% and 10% critical values respectively. For both statistics the rejection frequencies are close to 100%, which clearly outlines the importance of the error-correction mechanism for impulse response analysis.[10] The effect of the strength of the error-correction can be evaluated by comparing the benchmark parameter specification in columns 1 and 2 to columns 3 to 6 that report the simulation results with weaker degree of error correction. As expected, we can observe that the shares of FECM impulse responses and the rejection frequencies of the Olivei and Tenreyro test uniformly decrease with the strength of error-correction. These reductions, however, are not large, which indicates that even a relatively weak but genuine error-correction mechanism significantly affects impulse response analysis. The rejection frequencies of the Olivei and Tenreyro test marginally decrease with both N and T according to the CD statistic, and even slightly increase for the D statistic.

[9] Computation of relative mean-squared errors is based on 1000 Monte Carlo replications.

[10] As a robustness check, we considered some modifications to the basic data-generating process. In particular, the FECM in the simulation experiment contains 3 endogenous lags (uniform across equations), while the factors enter contemporaneously and with one lag. We repeated the same experiment also with one and three of both endogenous lags and lags of factors. The results, available upon request, are robust and fully in line with those presented in Table 1.

Panel B of Table 1 reports relative MSEs averaged over impulse responses of N variables to an identified permanent productivity shock. The FECM in general produces considerably lower MSEs, which can be largely attributed to the fact that the FAVAR impulse responses are biased due to omission of the error-corrections terms. With weaker error-correction strength relative MSEs increase uniformly across the impulse response horizons. The same is the effect of sample size, both along the time-series and cross-section dimensions, which is also in line with expectations.

Finally, Panel C of Table 1 reports the share of FECM responses outside the FAVAR confidence intervals. While this exercise does not take into proper account the uncertainty around the FECM responses, it provides additional evidence of the severe effects of omitting error correction terms in FAVARs. At short horizons over 78% of FECM responses lie outside the FAVAR 67% confidence intervals for the benchmark DGP specification, while the corresponding share for the 90% confidence interval is 56%. The shares increase with the horizon towards almost 100%. The effect of error-correction strength is again as expected: weaker error-correction results in smaller shares of FECM responses outside the FAVAR confidence intervals. The effect of sample size is not uniform across dimensions. Smaller T results in less pronounced differences between the models, while for a smaller N the differences appear to be more frequent.

Overall, this simulation experiment confirms the relevance of the inclusion of error correction terms in FAVAR models, suggesting that their omission can have sizeable effects, also in rather small panels and with error-correction mechanisms of moderate strength.

6 Empirical applications

In this section we illustrate the use of the FECM in identification of permanent productivity shocks using the identification scheme proposed in Section 3.3. The use of the FECM for structural analysis is illustrated further with an example of identification of monetary policy shocks using contemporaneous restrictions as in Bernanke et al. (2005). In both applications we focus on the empirical importance of the error-correction mechanism for the analysis of structural shocks.

We use a US dataset containing 128 monthly series over the period 1959:1 - 2014:7.[11] The source of data is FRED (see McCracken and Ng, 2015). In determining the order of integration of variables we follow McCracken and Ng (2015). The only exception are prices, for which we follow Bernanke et al. (2005) and treat them as I(1) instead of I(2). However, our main results about the importance of error-correction, available upon request, are robust to treating prices as I(2). Overall, the dataset contains 114 I(1) series

[11] The results in Bai and Ng (2006) for the use of estimated factors not to lead to a generated regressor bias requires N to be relatively large with respect to $\sqrt{T}$. In our dataset $T > N$ and $\sqrt{T}/N$ approximately 0.2. We use the dataset as it is, as representative of large panels of macroeconomic data. A detailed investigation of a potential generated regressor problem for our particular empirical analysis is beyond the scope of the paper, but given the dimensions of our dataset we think it is not likely to represent a significant problem.

and 14 I(0) series. Each series is also tested for the presence of a deterministic trend and, if detected, the series are detrended prior to estimating the FECM equations.[12]

The space spanned by F_t and G_t is estimated by the method of Bai and Ng (2004) that is consistent in presence of I(1) idiosyncratic components. Principal components are extracted from the data transformed to I(0) and then cumulated. The Bai and Ng (2002) criteria provided a relatively poor guide to selection of the total number of static factors q, signalling the maximum number in the search range. For comparability with our previous analysis with US data in Banerjee et al. (2014a) we set the total number of factors to 4. As a robustness check we replicated our analysis with up to 7 static factors. In all of the cases the Amengual and Watson (2007) approach indicates the number of dynamic factors to be equal to the number of static factors. Impulse response functions turn out to be broadly robust to the number of factors used in the analysis (see next Section 6.1 for more details and on-line Appendix B). Applying the Johansen trace test (Johansen, 1995) to system (19) indicates two non-stationary factors, i.e. $r_1 = 2$. This results is robust to the choice of the total number of factors r. Moreover, applying the Bai (2004) IPC2 information criterion to I(1) data also indicates $r_1 = 2$. Overall, these results indicate that working with two I(1) factors in our empirical application appears to be a sensible choice.

Our data contain both I(1) and I(0) variables, which we model in the following way. Denote by X_{it}^1 the I(1) variables and by X_{it}^2 the I(0) variables. The empirical FECM is then

$$\Delta X_{it}^1 = \alpha_i(X_{it-1}^1 - \Lambda_i F_{t-1}) + \Lambda_i^1(L)\Delta F_t + \Phi_i^1(L)G_t + \Gamma^1(L)\Delta X_{it-1}^1 + v_{it}^1 \tag{26}$$

$$X_{it}^2 = \Lambda_i^2(L)\Delta F_t + \Phi_i^2(L)G_t + \Gamma^2(L)\Delta X_{it-1}^2 + v_{it}^2 \tag{27}$$

The model for the I(1) variables in (26) is the FECM, while the model for the I(0) variables in (27) is a standard FAVAR. The FAVAR model does not contain the error-correction mechanism in the ΔX_{it}^1 equations

$$\Delta X_{it}^1 = \Lambda_i^1(L)\Delta F_t + \Phi_i^1(L)G_t + \Gamma^1(L)\Delta X_{it-1}^1 + v_{it}^1,$$

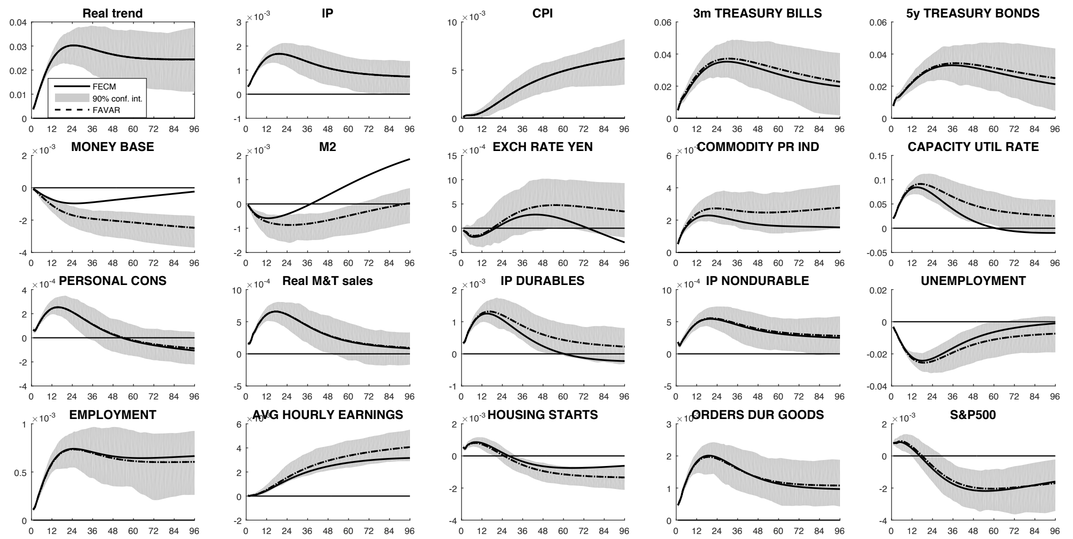
while the equations for X_{it}^2 are the same as those in (27).

The lag structure of the models is the following. Both the FAVAR model and the FECM contain three endogenous lags, while the factors enter contemporaneously and with one additional lag. This additional lag of factors serves to proxy for potentially omitted lags of X_j variables in equations for X_i, $i \neq j$. Robustness of the results has been checked by varying the number of endogenous lags from 1 to 6, and lags of factors from 0 to 3. Results turn out to be robust and are available upon request.

For the VAR for the factors we set the number of lags to 4 as indicated by the HQ

[12] The data are seasonally adjusted at source. In addition, we screened the data for outliers. Observations exceeding six times the inter-quartile range were removed and replaced by the median of 5 preceding observations.

Figure 1: Impulse responses to real stochastic trend in the US- FAVAR Vs FECM



and BIC information criteria. This resulted in a statistically well-specified model without residual autocorrelation.

To provide *prima facie* evidence of the importance of the error-correction terms in (26) we tested their significance with a standard t-test equation by equation. 77 and 91 out 114 equations have a statistically significant α_i at the 5% and 10% significance level respectively.

6.1 Impulse responses to permanent productivity shocks

We first present the analysis of structural permanent productivity shocks with corresponding impulse responses in Figure 1. The top left panel contains the response over time of the real permanent productivity trend.[13] Each sub-plot contains the impulse responses obtained with the FECM (solid line) and the FAVAR (dashed line) together with 90% bootstrapped confidence intervals of the FAVAR impulse responses.[14]

The impulse responses are broadly in line with economic theory. Along the adjustment path the real factor exhibits a hump-shaped response and after four years it levels off at the new higher steady state. Similar in shape are the positive responses of industrial production and measures of real private consumption and orders. Prices increase, which is in line with the DSGE evidence on the effect of permanent productivity improvements (Adolfson et al, 2007). The feature is exhibited also for other prices in the panel, but the corresponding impulse responses are not presented in Figure 1. Interest rates gradually

[13]Impulse responses for the remaining 107 variables of the panel are available upon request.

[14]The bootstrap procedure resamples factors and the observation equations as described in Section 5. In the construction of confidence intervals we follow Hall (1992) (see also Lütkepohl, 2005). We tried also bias correction as in Killian (1998), which resulted to perform rather poorly. The reason is the fact the the factor VAR in our case contains exactly r_1 unit roots, which are not preserved in general, which results in inconsistency of the bias correction procedure.

increase both in the short and longer end of the yield curve, reflecting an increase in real rates associated with an increase in productivity. The responses of money related variables are negative, but with quite pronounced differences between the FECM and the FAVAR. Consistently with higher interest rates, the dollar appreciates. Employment increases along the adjustment path, while unemployment rate temporarily decreases. The average wage rate also steadily increases in line with increased productivity. The positive effect on housing starts and the stock market index is only temporary, probably reflecting the effect of higher interest rates.

Figure B.1 in on-line Appendix B presents a robustness check of our results with respect of the number of estimated factors. In particular, we considered 4 to 7 factors. Results reveal a high degree of robustness of impulse responses of all major variables that measure output, demand, prices and labor market variables. Some heterogeneity is observed predominantly for financial variables.

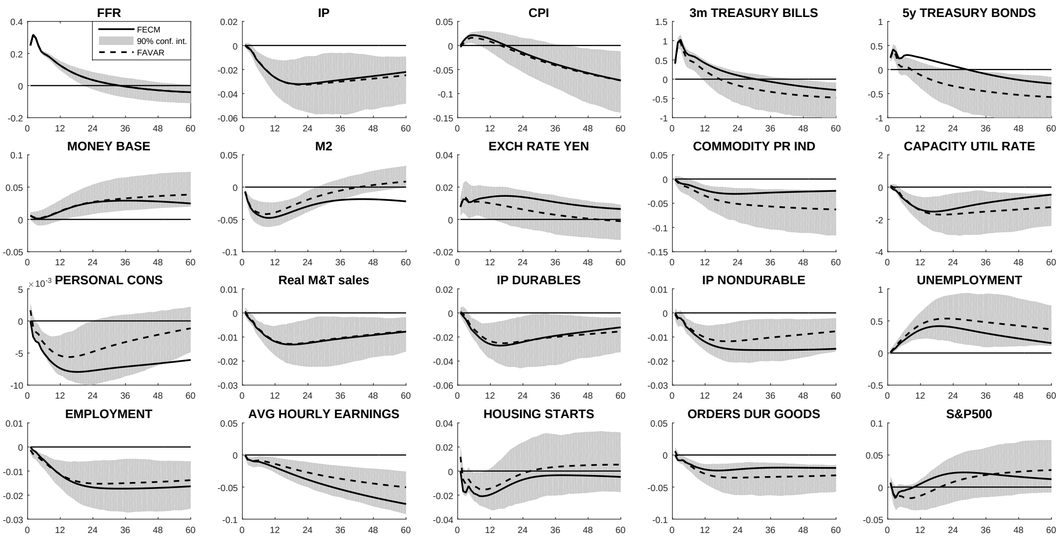
To facilitate a structural interpretation of the identified real permanent shocks, one can compare our impulse responses with the impulse responses reported by Adolfson et al. (2007) for an estimated DSGE model. The model of Adolfson et al. (2007) contains a stochastic productivity trend, which allows them to estimate the model on raw, non detrended data. Their impulse responses to a positive and permanent productivity shock (see Figure 5a in Adolfson et al., 2005) share a great degree of similarity with our impulse responses. The signs of responses are matched for most of the variables we report in Figure B.2 in on-line Appendix B. Measures of economic activity respond positively, as do prices and interest rates, wages increase and, finally, the real exchange rate appreciates. The stochastic trend response for the US case is different in its basic shape, namely, hump-shaped, but conditional on this feature, the adjusting dynamics of other variables are very comparable. Such direct comparability of basic shapes of the responses allows us to interpret the real permanent shocks identified with our approach as the structural permanent productivity shocks.[15]

6.2 Impulse responses to monetary policy shocks

The FECM can be used with other identification schemes. We illustrate this by replicating the analysis of Bernanke et al. (2005) in our framework. In particular, the federal funds rate is imposed as an observed factor in a 4-dimensional factor space. To account for the zero-lower bound, from mid-2009 the federal funds rate is replaced by the shadow short rate estimated by Wu and Xia (2016), who use it in an extension of the analysis of Bernanke et al. (2005). The identification of monetary policy shocks is obtained with a

[15]Barigozzi et al. (2016b) provide impulse responses of GDP, consumption and investment to a permanent real shock using quarterly US data and one I(1) factor. Their identification scheme differs from ours as it does not involve restrictions on long-run effects on observable variables. Their responses (presented with narrower, 68% confidence intervals) exhibit very pronounced hump-shaped responses reaching a peak one quarter after the shock, subsequently sharply decreasing and levelling off at a level lower than the initial change. Relative to the FECM such impulse responses show a smaller degree of similarity with the DSGE evidence.

Figure 2: Impulse responses to a monetary policy shock in the US - FAVAR Vs FECM



lower-triangular structure for the matrix H in (25), with the federal funds rate ordered last in the factor VAR (20).[16]

What we observe is coherence in terms of the basic shape of the impulse responses between the FAVAR and the FECM and in line with economic priors about the effects of a contractionary monetary policy shock. Industrial production, private consumption capacity utilization and employment decline, while the unemployment rate increases. Housing investment and manufacturing orders also decline, as expected from monetary contractions. The impulse response of the CPI exhibits an initial price puzzle, but prices eventually significantly and permanently decline. A decline in the wage rate is also in line with economic theory. We can also observe that the change in the federal funds rate strongly feeds into the yield curve, while the dollar appreciates.

6.3 Empirical relevance of the error-correction mechanism

From Figures 1 and 2 we can already observe that for some variables the impulse responses of the FECM and the FAVAR can be different, confirming the importance of error-correction mechanisms for impulse response analysis. This is especially evident for

[16] The purpose of presenting the analysis of monetary policy shocks is to provide an additional illustration of the importance of the error-correction mechanism for structural analysis with dynamic factor models. Our aim is not a discussion of schemes for identification of monetary policy shocks. The weakness of the Bernanke et al. (2005) scheme is that the contemporaneous restrictions are imposed on the latent factors and not observable variables. Forni and Gambetti (2010) overcome this deficiency and identify monetary policy shocks by imposing contemporaneous restrictions on a small number of observable variables. Barigozzi et al. (2016b), alternatively, identify monetary policy shocks by imposing sign restrictions on the contemporaneous responses of the federal funds rate, GDP and CPI.

the case of the monetary policy shock. The figures, however, report impulse responses for only a small subset of variables in the panel, and do not show the confidence bands for the FECM. Fuller account of the empirical effects of the error-correction terms is given in Table 2, which presents a summary of the results of the Olivei and Tenreyro (2010) test for impulse responses of all 114 variables for both the case of real permanent shock and the monetary policy shock. Entries to the Table are the percentages of variables within a given group of variables for which the impulse response obtained with the FECM are statistically differenct from those of the FAVAR at 10% significance level.

Across all 114 variables about 34% of impulse responses to a permanent productivity shock are significantly different according to the D statistic and 29% according to the CD statistic. For the case of the monetary policy shocks these shares are even higher, almost 50% and 43% respectively. Such test results clearly indicate the importance of the error-correction mechanism in impulse response analysis.

Table 2 also indicates that the importance of error-correction mechanism is not uniform across variables. In the case of the permanent productivity shock, above-average shares of statistically significant differences in impulse responses are detected for employment indicators, private consumption, monetary aggregates and wages. Little or no differences are detected for interest rates and housing variables. For output and prices about a quarter of responses are statistically different. Higher shares of statistically different responses of employment and wages are evidenced also in the case of the monetary policy shock. More importantly, however, we can observe substantial differences also for groups of variables of key interest in the analysis of the transmission mechanism of monetary policy: interest rates and prices indexes. For these two groups of variables the shares of significantly different responses exceeds 50%.

Table 2: Olivei and Tenreyro (2010) test of the difference between FECM and FAVAR impulse responses (share of statistically different response at 10% significance level (in %))

	Permanent productivity shock		Monetary policy shock	
Variables	D statistic	CD statistic	D statistic	CD statistic
All	34.2	28.9	49.6	43.4
Output	26.7	26.7	46.7	26.7
Employment	44.8	41.4	65.5	62.1
Consumption	80.0	50.0	20.0	20.0
Housing	14.3	14.3	28.6	28.6
Interest rates	0.0	0.0	62.5	62.5
Exchange Rates	25.0	25.0	25.0	25.0
Stock Prices	25.0	25.0	0.0	25.0
Money	38.5	38.5	38.5	23.1
Wages	33.3	33.3	100.0	100.0
Prices	25.0	15.0	60.0	50.0

7 Conclusions

In this paper we analyse the implications of cointegration for structural FAVAR models. Starting from a dynamic factor model for non-stationary data, we derive the factor-augmented error-correction model (FECM), its moving-average representation, and discuss estimation of the model parameters and of the impulse response functions, relying on the asymptotic theory developed in Bai and Ng (2004), which allows for some I(1) idiosyncratic components.

Structural analysis in the FECM can be conducted as in structural VARs. We provide the first analysis of long-run restrictions to identify a permanent productivity shock in the context of large cointegrated panels. Accounting for cointegration has important effects on the impulse responses to this shock as it reveals significant differences between the FECM and the FAVAR for about one third of the variables. Moreover, the FECM generates responses broadly in line with the theoretical DSGE analysis of, e.g., Adolfson et al. (2007). For the case of impulse responses to an identified monetary policy shock the share of significant differences between the FECM and the FAVAR is even higher.

The relevance of the error correction terms to avoid biases in FAVAR responses to shocks is also confirmed by means of simulations experiments. Simulation results show that the differences between the impulse response functions obtained by the FECM and the FAVAR are on average more pronounced the higher is the strength of the error-correction and the higher are the cross-section and the time series dimensions of the panel. Moreover, the differences in impulse responses are frequent also in samples of moderate size and with moderate strength of the error-correction mechanism.

Overall, these results suggest that the FECM that exploits the information in the levels of nonstationary variables to explicitly model cointegration provides an empirically important extension of classical FAVAR models for structural modelling. Other identification schemes such as sign restrictions could be also adopted in a FECM context. A detailed analysis of these is beyond the scope of this paper but provides an interesting topic for further research.

References

[1] Adolfson, M., S. Laseen, J. Linde and M. Villani (2007). Bayesian estimation of an open economy DSGE model with incomplete pass-through. *Journal of International Economics,* **72**, 481-511.

[2] Amengual, D. and M. W. Watson (2007). Consistent estimation of the number of dynamic factors in a large N and T Panel, *Journal of Business & Economic Statistics,* **25***,* 91-96.

[3] Anderson, T. W. (1951). Estimating linear restrictions on regression coefficients for multivariate normal distributions, *Annals of Mathematical Statistics*, 22, 327-51.

[4] Bai, J. (2004). Estimating cross-section common stochastic trends in nonstationary panel data. *Journal of Econometrics*, **122**, 137-183.

[5] Bai, J. and S. Ng (2002). Determining the number of factors in approximate factor models. *Econometrica*, **70**, 191-221.

[6] Bai, J. and S. Ng (2004). A PANIC attack on unit roots and cointegration. *Econometrica*, **72**, 1127-1177.

[7] Bai, J. and S. Ng, S. (2006). Confidence intervals for diffusion index forecasts with a large number of predictors and inference for factor-augmented regressions. *Econometrica*, **74**, 1133-1150.

[8] Bai, J. and S. Ng (2007). Determining the number of primitive shocks in factor models, *Journal of Business and Economic Statistics*, **25**, 52-60

[9] Bai, J., C. Kao and S. Ng (2009). Panel cointegration with global stochastic trends. *Journal of Econometrics*, **149**, 82-99.

[10] Banerjee, A. and M. Marcellino (2009). Factor-augmented error correction models, in J.L. Castle and N. Shephard, N. (eds.), *The Methodology and Practice of Econometrics - A Festschrift for David Hendry.* Oxford: Oxford University Press, 227-254.

[11] Banerjee, A., M. Marcellino and I. Masten (2014a). Forecasting with factor-augmented error correction models. *International Journal of Forecasting*, **30,** 589-612.

[12] Banerjee, A., M. Marcellino and I. Masten (2014b). Structural FECM: Cointegration in large-scale structural FAVAR models. CEPR Discussion Papers Series No. 9858.

[13] Barigozzi, M., M. Lippi and M. Luciani (2016a). Dynamic Factor Models, Cointegration, and Error Correction Mechanisms, Finance and Economics Discussion Series 2016-018. Washington: Board of Governors of the Federal Reserve System.

[14] Barigozzi, M., M. Lippi and M. Luciani (2016b). Non-Stationary Dynamic Factor Models for Large Datasets," Finance and Economics Discussion Series 2016-024. Washington: Board of Governors of the Federal Reserve System.

[15] Bernanke, B.S., J. Boivin and P. Eliasz (2005). Measuring the effects of monetary policy: a factor-augmented vector autoregressive (FAVAR) approach. *Quarterly Journal of Economics*, **120**, 387-422.

[16] Blanchard, O. J. and D. Quah (1989). The dynamic effects of aggregate demand and supply disturbances, *American Economic Review*, **79**, 655-73.

[17] Choi, I. (2011). Efficient estimation of nonstationary factor models. RIME Working paper No. 2011-13.

[18] Eickmeier, S. (2009). Comovements and heterogeneity in the euro area analyzed in a non-stationary dynamic factor model. *Journal of Applied Econometrics*, **24**, 933-959.

[19] Forni, M., Gambetti, L. and Sala, L. (2014). No news in business cycles. *The Economic Journal*, **124**, 1168-1191.

[20] Forni, M. and L. Gambetti (2010). The dynamic effects of monetary policy: A structural factor model approach, *Journal of Monetary Economics*, **57**, 203-216.

[21] Forni, M., M. Hallin, M. Lippi and L. Reichlin (2000). The generalized dynamic-factor model: Identification and estimation. *Review of Economics and Statistics*, **82**, 540-554.

[22] Forni, M., D. Giannone, M. Lippi and L. Reichlin (2009). Opening the black box: Structural factors models with large cross sections. *Econometric Theory,* **25**, 1319-1347.

[23] Gengenbach, C., Urbain, J.-P., and J. Westerlund (2015). Error correction testing in panels with common stochastic trends. *Journal of Applied Econometrics*, doi: 10.1002/jae.2475.

[24] Hall, P. (1992). *The Bootstrap and Edgeworth Expansion*, Springer, New York.

[25] Johansen, S. (1995). *Likelihood-based inference in cointegrated vector autoregressive models*. Oxford University Press, Oxford and New York.

[26] King, R. G., C.I. Plosser, J.H. Stock and M. W. Watson (1991). Stochastic trends and economic fluctuations, *American Economic Review*, **81,** 819-40.

[27] Lütkepohl, H. (2005). *New Introduction to Multiple Time Series Analysis*. Springer, New York.

[28] Lütkepohl, H. (2014). Structural vector autoregressive analysis in a data-rich environment: A Survey. DIW Discussion paper 1351.

[29] McCracken, M. W. and S. Ng (2015). FRED-MD: A monthly database for macroeconomic research, Working Papers 2015-12, Federal Reserve Bank of St. Louis.

[30] Olivei, G. and S. Tenreyro (2010). Wage-setting patterns and monetary policy: international evidence. *Journal of Monetary Economics*, **57,** 785-802.

[31] Stock, J.H. and M.W. Watson (2002a). Forecasting using principal components from a large number of predictors, *Journal of the American Statistical Association*, **97**, 1167-1179.

[32] Stock, J.H. and M.W. Watson (2002b). Macroeconomic forecasting using diffusion indexes. *Journal of Business and Economic Statistics*, **20**, 147-162.

[33] Stock, J.H. and M.W. Watson (2005). Implications of dynamic factor models for VAR analysis. NBER Working Paper 11467.

[34] Vlaar, P. J. G. (2004). On the asymptotic distribution of impulse response functions with long-run restrictions, *Econometric Theory,* **20**, 891.

[35] Warne, A. (1993). A common trends model: identification, estimation and inference. University of Stockholm, IIES Seminar Paper No. 555.

[36] Westerlund, J. and R. Larsson (2009). A note on the pooling of individual PANIC unit root tests. *Econometric Theory* **25**, 1851-1868.

[37] Wu, J. C. and F. D. Xia (2016). Measuring the macroeconomic impact of monetary policy at the zero lower bound, *Journal of Money, Credit, and Banking*, **48**, 253-291.